AD/A-005 906

ON A SIMPLE RESOURCE-VALUE TRANSFER ECONOMY

A. Charnes, et al

Texas University at Austin

Prepared for:

Office of Naval Research
Aston University, England

July 1974

Unclassified

Security Classification

AD-[illegible] 706

DOCUMENT CONTROL DATA - R & D

1. ORIGINATING ACTIVITY (Corporate author) Center for Cybernetic Studies The University of Texas	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP
3. REPORT TITLE On A Simple Resource-Value Transfer Economy	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)	
5. AUTHOR(S) (First name, middle initial, last name) A. Charnes S. Littlechild J. Rousseau	
6. REPORT DATE July 1974	7a. TOTAL NO. OF PAGES: 21 / 7b. NO. OF REFS: 0
8a. CONTRACT OR GRANT NO. N00014-67-A-0126-0008; 0009 b. PROJECT NO. NR 047-021	9a. ORIGINATOR'S REPORT NUMBER(S) Center for Cybernetic Studies Research Report CCS 205 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited.	
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Office of Naval Research (Code 434) Washington, D. C.

13. ABSTRACT

A five-resource, three-good, five-owner-consumer group example of a resource-value transfer economy under the equilibrium interpretation adduced by Charnes and Cooper is studied. A reinterpretation of owner-consumer group size as an index of standard of living is made. The results on the hypothetical example indicate that substantial changes in standard of living, etc. can result from relatively minor changes in total resource valuation if these changes are in critical resources. They further suggest that it may be desirable to extend the model via the Charnes-Cooper extremal principle to better account for relationships between levels of industrial activity and population group sizes.

(24)

Unclassified
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Resource-Value Transfer Economy						
Extremal principle						
Extended geometric programming						

DD FORM 1 NOV 65 1473 (BACK)
S/N 0102-014-6600

Unclassified
Security Classification

Research Report CCS 205

ON A SIMPLE RESOURCE-VALUE TRANSFER ECONOMY

by

A. Charnes
S. Littlechild*
J. Rousseau

July 1974

*University of Aston Management Centre, 36 Wake Green Road, Moseley, Birmingham, England.

This research was partly supported by Project No. NR 047-021, ONR Contracts N00014-67-A-0126-0008 and N00014-67-A-0126-0009 with the Center for Cybernetic Studies, The University of Texas. Reproduction in whole or in part is permitted for any purpose of the United States Government.

CENTER FOR CYBERNETIC STUDIES

A. Charnes, Director
Business-Economics Building, 512
The University of Texas
Austin, Texas 78712
(512) 471-1821

16

ABSTRACT

A five-resource, three-good, five-owner-consumer group example of a resource-value transfer economy under the equilibrium interpretation adduced by Charnes and Cooper is studied. A reinterpretation of owner-consumer group size as an index of standard of living is made. The results on the hypothetical example indicate that substantial changes in standard of living, etc. can result from relatively minor changes in total resource valuation if these changes are in critical resources. They further suggest that it may be desirable to extend the model via the Charnes-Cooper extremal principle to better account for relationships between levels of industrial activity and population group sizes.

I. Introduction

In a recent paper (Rendiconti di Accademia Nazionale dei Lincei, April 1971) entitled "An Extremal Principle for Accounting Balance of a Resource Value-Transfer Economy: Existence, Uniqueness and Computation" Professors Charnes and Cooper characterized by an extremal principle an economy (due to W.P. Drews) consisting of r resources and s "resource-owner" groups of various "sizes" (owner "sizes" will be interpreted differently later) which are in possession of the resources. Each resource may be used in each of n industrial activities to produce m final goods which are consumed by the "resource-owner" (consumer) groups. In this "economy," agreed upon monetary values of resources are transferred into agreed upon monetary receipts of the owner groups. Prices of resources and "sizes" of owner groups are adjusted so that (1) total value of resources transferred equals total monetary receipts, (2) ("Accounting Balance"). Consumer goods prices and industrial activity levels are such that (a) the value of each resource equals the total sum spent on it, (b) receipts equal expenditure for each owner group. These notes are an attempt to apply this model to a simplified economy in which there are five resources (r=5), five owner (consumer) groups (s=5), four industrial activities (n=4) and three final goods (m=3).

II. Economy Constructs

Before developing our example we give a brief resume of the economy as elucidated by Charnes and Cooper.

Following their notation, let

$v_1, \ldots, v_r$ denote the value of resources, and

$\delta_1, \ldots, \delta_s$ the receipts or incomes of the owner (consumer) groups, where

$$\gamma_i, \delta_j > 0 \qquad \forall i, j$$

R, N and C are three nonnegative matrices:

$R(r \times n)$: resources required per unit industrial activity

$N(n \times m)$: units of industrial activity required to produce one unit of final good

$C(m \times s)$: units of final goods consumed per unit "size" of owner (consumer) group

Further,

p^T denotes the row vector of unit prices of resources,

q the column vector of "sizes" of owner (consumer) groups,

y^T the row vector of final goods prices

x the column vector of industrial activity levels.

For balance it is required that consumer goods unit prices match unit costs of production

(1) $$y^T = p^T RN$$

levels of production be adequate to meet demand

(2) $$x = NCq$$

the value of each resource be recouped from the total sum spent on it

(3)' $$\gamma_i = p_{ii} R x \qquad i = 1, \ldots, r$$

group receipts cover (match) expenditure for each owner (consumer) group

(4)' $$\delta_j = y^T C_j q_j \qquad j = 1, \ldots, s$$

letting M=RNC and substituting for x and y^T we obtain

(3) $$\gamma_i = p_i({}_iMq) \qquad i = 1, \ldots, r$$

(4) $$\delta_j = (p^T M_j)q_j \qquad j = 1, \ldots, s$$

The reader is referred to Charnes and Cooper's paper for the characterization of the existence and uniqueness of equilibrium and for possible computational procedures.

III. Interpretation and An Application

In this section we give an interpretation of the model outlined above and describe an (initial) equilibrium situation. In the following section we introduce disturbances into the economy which result in the attainment of new equilibrium positions.

The example chosen here is kept deliberately small and uncomplicated to enable direct hand computation. Specifically we take the resources employed to be labor, capital, land, entrepreneurial expertise and governmental activity. Prices of these resources are then interpreted as wages, interest, rent, salaries and taxes respectively.

Industrial activities are taken to be agriculture, industry, banking and retailing. From elements of the resource matrix $[R_{ik}]$ and activity levels x_k the quantity of each resource used can be obtained in an obvious way. Multiplication by the price of the resource immediately gives the total sum spent on the resource.

Consumption is assumed to be carried out by households, where the role "bread-winner" of each household may be categorized as a farmer (i.e. a farm owner), a merchant, a skilled (white-collar) worker, an unskilled (blue-collar) worker or a retired person. We further assume that the population consists of 100 million households with the percentage number of households in each category being 2, 12, 30, 40 and 16 respectively.

A final good is considered to be one of three general types: a durable good, a nondurable good or a service. A unit of each type of good is interpreted as a "bundle" of various goods of that particular kind and the unit price of each type of good is then the purchase price of such a bundle. Each consumer group would then purchase so many units of each type of good per time period, say per year.

In order to correctly interpret the consumption matrix $[C_{gi}]$ we introduce the notion of "standard of living." Unit standard of living is defined here as a basic consumption pattern of goods by consumer groups. We take as our reference point the consumption by an unskilled worker: in order for an unskilled worker to attain unit standard of living we assume that he consumes one unit of each type of good per year, i.e. every year he will purchase one "bundle" of durable goods, one "bundle" of nondurable goods and one "bundle" of services. In general, however, other groups will have different consumption behaviors and at unit standard of living will purchase quantities of each type of good proportional to that consumed by our unskilled worker. These suggested proportions are set out as matrix C^0 in Table I.

If we multiply these proportions by the number of households in each group we will obtain the (group) consumption matrix C which shows total consumption of goods by groups at unit standard of living. By incorporating consumer goods prices we can obtain the expenditure of each group (and hence the income required to sustain each group) at unit standard of living. However, in general, no group need actually be at unit standard of living. To allow for this we interpret group "sizes" q as measures of the living standard actually enjoyed. In other words, for a particular group j, q_j is the number of unit standards of living actually enjoyed by group j. If q_j for unskilled labor is equal to 2 then this means each unskilled laborer actually purchases twice as much per year as he would do at unit standard of living.

Multiplying the jth group's expenditure at unit living standard by the appropriate q_j will then give the actual expenditure of group j (and hence the income required to maintain group j) at standard of living represented by q_j. We note here that farmers and merchants would receive income in the form of profits through their business activities, skilled and unskilled workers receive salaries and wages from their employers and retired persons receive transfer payments (pensions) from the government.

In the extremal principle developed by Charnes and Cooper, for prespecified (arbitrary) values of resources (v_i) and owner (consumer) group incomes (δ_j) and given matrices R, N and C there exists a unique set of relative resource prices (p), consumer goods prices (y), activity levels (x), and consumer group "sizes" (standard of living indices) (q) consistent with the equilibrium conditions stated earlier and the "transfer matrix," M, conditions given in the Charnes-Cooper paper. We now give specific values for these parameters and variables that result in just such an equilibrium position (which we hereafter call the initial situation). We make one additional simplification, however, we assume that all groups are at their unit standard of living (i.e. $q_j = 1 \ \forall j$).

In the following section we relax this assumption. Suggested values for the elements of the resource matrix R, the activity matrix N, consumption matrix C and matrix C^o are given in table I. Let us assume the economy has a Net National Product of $1,202,000 million. Table II shows how this amount is allocated to each resource and each consumer group's income level. We note that total value of resources equals total income of the owner (consumer) groups and is in "accounting balance." Table III gives the equilibrium set of resource prices and quantities of resources used (and hence total expenditure on each resource) while Table IV contains the corresponding levels of industrial activity and consumer goods prices. The standard of living indices and expenditures by groups and households are given in Table V.

TABLE I

RESOURCE MATRIX $[R_{ik}]$

i \ k	Agric.	Ind.	Bank.	Ret.
Lab.	5	20	5	1
Cap.	4	10	8	2
Land	200	0	0	0
Ent.	0	0	0	1
Govt.	7	22	7	2

ACTIVITY MATRIX $[N_{kg}]$

k \ g	Durables	Nondurables	Services
Agric.	0.00	0.01	0.00
Ind.	0.03	0.01	0.02
Bank.	0.01	0.01	0.01
Ret.	0.01	0.01	0.00

$[C^o_{gj}]$

g \ j	Farm	Mer.	S. Labor	U. Labor	Ret. Per.
Durables	3	2.00	1.5	1.0	0.25
Nondurables	2	1.75	1.5	1.0	0.5
Services	3	2.00	1.5	1.0	0.5
# Households (Millions)	2	12	30	40	16

TABLE I (Cont.)

CONSUMPTION MATRIX (C_{gj})
(Millions of units)

g \ j	Farm	Mer.	S. Labor	U. Labor	Ret. Per.
Durables	6	24	45	40	4
Nondurables	4	21	45	40	8
Services	6	24	45	40	[illegible]

TABLE II

Resource i	Value of Resource v_i $ Millions	Owner (Consumer) Group j	Income ξ_j $ Millions
Labor	852,350	Farmers	55,220
Capital	110,360	Merchants	233,130
Land	23,600	Skilled labor	450,900
Ent.	18,960	Unskilled labor	400,800
Govt.	196,820	Retired Persons	62,040
	$\sum_i v_i = 1,202,090$		$\sum_j \xi_j = 1,202,090$

TABLE III

Resource i	Resource Price p_i	Resources Used ${}_iRx$	Expenditure on Resource $p_{ii}R_x$ ($ millions)
Labor	$5000/man-year	170.47 million man year	832,350
Capital	$1000/$10000-year	$1103.6 b. of capital	110,360
Land	$100/acre-year	236 m. acres	23,600
Ent	$8000/man-year	2.37 m. entrepreneurs	18,960
Govt.	$1000/$4000-year	$787.28 b. taxable profits	196,820

TABLE IV

Industrial Activity k	Activity Level x_k (millions of units)	Consumer Good g	Consumer Good Price y_g ($ per year)
Agriculture	1.18	Durables	4530
Industry	7.21	Nondurables	2450
Banking	3.60	Services	3040
Retailing	2.37		

TABLE \

Consumer group j	Annual Expenditure at unit standard of living $y^T c_j$ ($ millions)	Index of living standard q_j	Actual Expenditure $y^T c_j q_j$ ($ million)	# of Households (millions)	Expenditure per household ($000s)
Farmers	55,220	1.0	55,220	2	27,610.0
Merchants	233,130	1.0	233,130	12	19,427.5
Skilled labor	450,900	1.0	450,900	30	15,030.0
Unskilled labor	400,800	1.0	400,800	40	10,020.0
Retired persons	62,040	1.0	62,040	16	3,877.5

IV. Disturbance from Equilibrium

Consider now a disturbance resulting in a move away from the equilibrium situation of section III. Specifically, let us suppose that there is a 10% increase in Net National Product brought about by a 10% increase in the use of capital. Further, we assume that funds are allocated to resources in the same proportions as before and so expenditure on each resource is now increased by 10%.

Let us consider two possible situations. In the first case each consumer group finds its income increased by 10%, thus totally accounting for the increase in NNP. In the second case we look at the possibility of a single group benefiting from the increase.

(a) All consumer groups benefit proportionally.

The new resource expenditures and group incomes are given in Table VI below. Each individual of each group finds his income increased by 10%.

TABLE VI

Resource i	Value of Resource v_i \$ millions	Owner (Consumer) Group j	Income δ_j \$ Millions
Labor	937,585	Farmers	60,742
Capital	121,396	Merchants	256,443
Land	25,960	Skilled labor	495,990
Ent.	20,856	Unskilled labor	440,880
Govt.	216,502	Retired Persons	68,244
	$\sum_i v_i = 1,322,299$		$\sum_j \delta_j = 1,322,299$

If resource prices are held constant then consumer goods prices are likewise unaltered. However all industrial activity levels (and hence quantities of resources used) and all standards of living will increase by 10%.

The details are set out below in Table VII.

TABLE VII

Industrial Activity k	Activity Level λ_k
Agriculture	1.293
Industry	7.931
Banking	3.960
Retailing	2.607

Resource i	Resources Used $_i Rx$	Consumer Group j	Std. of living index q_j
Labor	187.517 m. man-yrs.	Farmers	1.1
Capital	$1213.96 b. capital	Merchants	1.1
Land	259.6 m. acres	Skilled Labor	1.1
Ent	2.607 m. entrepreneurs	Unskilled Labor	1.1
Govt.	$866.008 b. taxable profits	Ret. Persons	1.1

<u>Note</u>

Consider the following equilibrium equations from Charnes and Cooper's paper.

(1) $y^T = p^T RN$

(2) $x = NCq$

(3') $v_i = p_{ii} Rx$ $\quad i = 1, \ldots, r$

(4') $\delta_j = y^T C_{jj} q_j$ $\quad j = 1, \ldots, s$

For fixed v_i $(i = 1, \ldots, 4)$ and δ_j $(j = 1, \ldots, s)$, a reduction in resource prices p and goods prices y will require increases of the same magnitude in activity levels x and standard of living q. With no additional constraints on activity levels and resource availability then, for fixed NNP, it would appear that consumers could be made arbitrarily well off as p and y tend to zero.

(b) <u>Increase in NNP absorbed by one consumer group</u>

Assume now that due to strong trade union bargaining the entire increase in NNP is absorbed by unskilled labor. In this case all incomes are held constant except that of unskilled labor. The new resource values and group incomes are set out in Table VIII.

TABLE VIII

Resource i	Value of Resource v_i ($ millions)	Owner Group j	Income ξ_j ($ millions)
Labor	937,585	Farmers	55,220
Capital	121,396	Merchants	233,130
Land	25,960	Skilled Labor	150,900
Ent.	20,856	Unskilled Labor	521,009
Govt.	216,502	Ret. Persons	62,040
	$\sum_i v_i = 1,322,299$		$\sum_j \xi_j = 1,322,299$

Note from the formulation of the model that

(3) $v_i = p_i ({}_iMq) \quad i = 1, \ldots, 4$

(4) $\xi_j = (p^T M_j) q_j \quad j = 1, \ldots, s$

where $M \equiv RNC$

We make use of these relationships to solve for the new values of p and q by an iterative process in which a series of p and q vectors are obtained which converge to the new equilibrium p and q vectors. Letting superscript o denote the initial equilibrium situation described in the previous section, and superscript ^ denote the fixed values for v and ξ as given above, then we have

$$q_j^1 = \frac{\hat{\delta}_j}{p^{o\,T} M_j} \quad j = 1, \ldots, 5, \text{ and } p_i^1 = \frac{\hat{v}_i}{{}_i M q^1} \quad i = 1, \ldots, 5$$

$$q_j^2 = \frac{\hat{\delta}_j}{p^{1\,T} M_j} \quad j = 1, \ldots, 5, \text{ and } p_i^2 = \frac{\hat{v}_i}{{}_i M q^2} \quad i = 1, \ldots, 5$$

in general

$$q_j^{k+1} = \frac{\hat{\delta}_j}{p^{k\,T} M_j} \quad j = 1, \ldots, 5, \text{ and } p_i^{k+1} = \frac{\hat{v}_i}{{}_i M q^{k+1}} \quad i = 1, \ldots, 5$$

Having obtained new equilibrium p and q vectors we can obtain the corresponding x and v vectors from

(1) $v^T = p^T RN$, and

(2) $x = NCq$

Some criteria for successful convergence must obviously be established in order to know when to stop the iterative procedure, but for our simple example a good approximation to equilibrium was achieved after two iterations. The p and q vectors obtained were

$$q' = \begin{bmatrix} 1.00001 \\ 1.00002 \\ 1.000024 \\ 1.29995 \\ 1.00003 \end{bmatrix} \quad \text{and} \quad p' = \begin{bmatrix} 5000.187 \\ 999.967 \\ 99.846 \\ 7990.805 \\ 1000.009 \end{bmatrix}$$

We approximate them by

$$q = \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.3 \\ 1.0 \end{bmatrix} \quad \text{and} \quad p = \begin{bmatrix} 5000 \\ 1000 \\ 100 \\ 7991 \\ 1000 \end{bmatrix}$$

and obtained corresponding x and y vectors

$$x = \begin{bmatrix} 1.30 \\ 7.93 \\ 3.96 \\ 2.61 \end{bmatrix} \qquad \text{and} \qquad y = \begin{bmatrix} 4529.91 \\ 2449.91 \\ 3049.00 \end{bmatrix}$$

Because we had used (close) approximate values for the elements of p and q we checked back on the values of v_i and ξ_j which could be obtained using

(3') $v_i = p_{ii} Bx \qquad i = 1, \ldots, 5$

(4') $\xi_j = y^T C_j q_j \qquad j = 1, \ldots, 5$

The values we obtained are set out below in Table IX and show a good degree of correspondence with the fixed values given in Table VIII. The new equilibrium values are then collected in Tables X and XI and XII.

TABLE IX

Resource	Value of Resource v_i (\$ millions)	Owner Group j	Income ξ_j (\$ millions)
Labor	937,550	Farmer	55,219
Capital	121,390	Merchants	233,126
Land	26,000	Skilled Labor	450,892
Ent.	20,856	Unskilled Labor	521,031
Govt.	216,500	Ret. Persons	62,039
	$\sum_i v_i = 1,322,296$		$\sum_j \xi_j = 1,322,307$

TABLE X

Resource i	Resource Price p_i	Resources Used ${}_iRx$	Expenditure on Resource $p_i\ {}_iRx$ ($ millions)
Labor	$5000/man-year	187.51 million man-year	937,550
Capital	$1000/$10000-year	$1213.9 b. of capital	121,390
Land	$100/acre-year	260 m. acres	26,000
Ent.	$7991/man-year	2.61 m. entrepreneurs	20,856
Govt.	$1000/$4000-year	$866 b. taxable profits	216,500

TABLE XI

Industrial Activity k	Activity Level x_k (millions of units)	Consumer Good g	Consumer Good Price y_g ($ 000s)
Agriculture	1.30	Durables	4529.91
Industry	7.93	Nondurables	2449.91
Banking	3.96	Service	3040.00
Retailing	2.61		

TABLE XI (Cont.)

Consumer Group	Annual Expenditure at unit standard of living $v^T C_j$ ($ millions)	Index of living standard q_j	Annual Expenditure $v^T C_j q_j$	# of Households (millions)	Annual Exp. per household ($000s)
Farmers	55,219	1.0	55,219	2	27,609.5
Merchants	233,126	1.0	233,126	12	19,427.2
Skilled labor	450,892	1.0	450,892	30	15,029.7
Unskilled labor	400,793	1.3	521,031	40	13,025.8
Ret. Persons	62,039	1.0	62,039	16	3,877.5

As it so happened in our example consumer goods prices and resource prices did not change significantly, so in order for unskilled labor to enjoy a 30% increase in their living standard the 10% increase in expenditure on resources (the 10% rise in NNP) resulted in greater quantities of resources being used and consequently higher industrial activity levels being required, each by a factor of 10%.

Note that in the initial equilibrium situation the work force needed consisted of 170 million laborers and over 2 million entrepreneurs, and in the second (case b) equilibrium position, 187 million laborers and over 2-1/2 million entrepreneurs were required. Yet we assumed only 100 million households with single "bread-winners"! One could interpret these results as meaning that multiple bread-winners per household are required for the equilibrium. In any event, they show that the extremal models should be extended to automatically take care of constraints on the availability of labor, on availability of resources and on productive capacity. Some such extensions have already been formulated by Charnes and Cooper.

An alternative type of disturbance is presently under investigation. What is the effect of changes in the values of elements of, say, the activity matrix N brought about by changing technology? The results for this analysis are not yet complete, either.